

# Novel vortex lattice transition in $d$ -wave superconductors

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## Abstract

We study the vortex state in a magnetic field parallel to the  $c$  axis in the framework of the extended Ginzburg Landau equation. We find the vortex acquires a fourfold modulation proportional to  $\cos(4\phi)$  where  $\phi$  is the angle  $\mathbf{r}$  makes with the  $a$ -axis. This term gives rise to an attractive interaction between two vortices when they are aligned parallel to  $(1, 1, 0)$  or  $(1, -1, 0)$ . We predict the first order vortex lattice transition at  $B = H_{cr} \sim \kappa^{-1} H_{c2}(t)$  from triangular into the square lattice tilted by  $45^\circ$  from the  $a$  axis. This gives the critical field  $H_{cr}$  a few Tesla for YBCO and Bi2212 monocrystals at low temperatures ( $T \leq 10K$ ).

The  $d$ -wave superconductivity in the hole-doped high  $T_c$  cuprate superconductors now appears to be universally established [1,2]. Then in a magnetic field parallel to the  $c$  axis we expect a square vortex lattice tilted  $45^\circ$  from the  $a$  axis at least in the vicinity of  $B = H_{c2}(T)$  and below  $t(\equiv T/T_c) \leq 0.81$  [3,4]. Indeed such vortex lattices though elongated in the  $a$  direction have been seen by small angle neutron scattering (SANS) [5] and scanning tunnelling microscope (STM) [6] both in twinned YBCO monocrystals at low temperature ( $T \sim 4K$ ) and in a small magnetic field ( $B \sim$  a few Tesla). The object of this letter is to determine theoretically the corresponding vortex lattice transition at a small magnetic field which is experimentally relevant. Also in order this vortex lattice transition is to be observable experimentally, it should take place in a magnetic field smaller than the one where vortex lattice melting is established [7,8]. Further we shall show that the transition is exothermal and of the first order. Earlier the related transition has been analyzed in terms of the generalized London equation, which may be obtained from the phenomenological Ginzburg Landau equation for  $d$ -wave superconductivity [9] or from the rhombic term arising from the normal state properties [10]. However these authors concentrated on the modification of the magnetic interaction energy due to the quartic term in the kinetic energy. We believe, on the contrary, that the important modification lies in the vortex core structure [11]. Nevertheless these authors obtained the vortex lattice transition from a triangular lattice in small magnetic field to a rhombic lattice in a higher field. The result is similar to ours except for two crucial differences. First we find that the critical field  $H_{cr}$  where this transition takes place behaves like  $\kappa^{-1}H_{c2}(t)$  where  $\kappa$  is the Ginzburg Landau parameter and  $H_{c2}(t)$  the upper critical field. This means  $H_{cr} \sim 10^{-2}H_{c2}(t)$  in high  $T_c$  cuprates. Second the transition is of the first order with a sharp negative divergence in the magnetization.

Within the weak-coupling model for  $d$ -wave superconductor [12] the extended Ginzburg-Landau equation is given by [11]

$$\left(1 + (\partial_x^2 + \partial_y^2) + \epsilon \left[5(\partial_x^2 + \partial_y^2)^2 + 2(\partial_x^2 - \partial_y^2)^2\right]\right) f(\mathbf{r}) = |f(\mathbf{r})|^2 f(\mathbf{r}). \quad (1)$$

where we introduces the reduced unit  $\mathbf{r} \rightarrow \mathbf{r}/\xi(T)$ ,  $\Delta(\mathbf{r}) = \Delta(T)f(\mathbf{r})$ ,

$$\xi(T)^2 = \frac{7\zeta(3)v^2}{2(4\pi T)^2(-\ln t)}, \quad \Delta(T)^2 = \frac{(4\pi T)^2(-\ln t)}{21\zeta(3)}, \quad (2)$$

and

$$\epsilon \equiv 31\zeta(5)(-\ln t)/196\zeta(3)^2 \sim 0.114(-\ln t), \quad (3)$$

where  $t = T/T_c$ . Here  $\partial_x$  and  $\partial_y$  are the gauge invariant differential operators. Eq.(1) has been obtained previously by Enomoto et al. [13], though we suppressed all the other terms of the order of  $(-\ln t)$ , which is not necessary for our purpose. Due to the quartic terms in Eq.(1) the single vortex solution is now given by [11]

$$f(\mathbf{r}) = g(r)e^{i\phi} + \epsilon \left( e^{4i\phi}\alpha(r) + e^{-4i\phi}\beta(r) + \gamma(r) \right) e^{i\phi}. \quad (4)$$

where  $g(r) \sim \tanh r$  and  $|f(\mathbf{r})|$  and the phase of  $f(\mathbf{r})$  are shown in Fig.1 and Fig.2. For clarity we have chosen  $\epsilon = 1$ . The ridges in  $|f(\mathbf{r})|$  running along four diagonals are clearly seen, which are crucial for the fourfold core interaction energy. Also the phase distortion propagates far away from the vortex center. For  $r \gg 1$ ,  $\alpha(r)$  and  $\beta(r)$  are obtained as [11]

$$\alpha(r) = \frac{5}{2}r^{-2} + \left(\frac{5}{4} - \frac{55}{4}\log r\right)r^{-4} + \dots, \quad (5)$$

$$\beta(r) = -\frac{5}{2}r^{-2} + \left(\frac{5}{4} + \frac{55}{4}\log r\right)r^{-4} + \dots, \quad (6)$$

which tell us the explicit form of the asymptotics. This result is consistent with Enomoto et al. [13]. In the presence of the fourfold distortion around single vortex, this term modifies drastically interaction between two vortices. Now let us consider the vortex lattice. Within the dimensionless unit the free energy of the vortex lattice is given by;

$$\Omega = \int d^2r \left( -\frac{1}{2}|\Delta|^4 + \frac{1}{8\pi}b^2 \right), \quad (7)$$

where  $b(\mathbf{r})$  is the local induction. Making use of the usual approximation for  $\Delta(\mathbf{r}) = \Delta \prod_i f(\mathbf{r} - \mathbf{r}_i)$  where  $\mathbf{r}_i$  is the position of the  $i$ -th vortex. We are here interested in the region  $1 \ll d \ll \kappa$  (or  $\xi(T) \ll d \ll \lambda(T)$  in the natural unit), where  $d$  is the distance between 2 neighboring vortices. Then neglecting terms exponentially small we obtain

$$\Omega = -\frac{1}{2} \left( A - a_1 \xi^2 n_\phi - \epsilon 10 a_1 \xi^2 n_\phi \sum'_{l,m} \frac{\xi^4}{r_{l,m}^4} \cos 4\theta_{l,m} \right) + \frac{2\pi}{\kappa^2} n_\phi \xi^2 \sum'_{l,m} K_0 \left( \frac{r_{l,m}}{\lambda} \right), \quad (8)$$

where  $A$  is the area,  $a_1 = \frac{8\pi}{3}(\ln 2 + \frac{1}{8}) \simeq 6.354$ ,  $n_\phi = B/\phi_0$  the vortex density per unit area and  $\mathbf{r}_{l,m} = 2ld(\cos \theta, \sin \theta) + 2md(\cos \theta, -\sin \theta)$  or  $(2l+1)d(\cos \theta, \sin \theta) + (2m+1)d(\cos \theta, -\sin \theta)$ . Here we limit ourselves to the vortex lattice with isocles unit cell. Also as to the magnetic interaction we consider here only the isotropic term  $\kappa^{-2}K_0(\frac{r}{\lambda(T)})$  since the anisotropic correction is much smaller than the vortex core interaction at least when  $\kappa \gg 1$ . The last term in Eq.(8) is handled using the Poisson transform as in [14]. Finally we transform Eq.(8) as

$$\Omega = \Omega_0 + \frac{2\pi\xi^2 H_{cr}}{\kappa^2 \phi_0} \bar{f} \left( \frac{B}{H_{cr}} \right), \quad (9)$$

where

$$\Omega_0 = -\frac{A}{2} + \frac{a_1 \xi^2}{2\phi_0} B + \frac{2\pi \xi^2}{\kappa^2 \phi_0} B \left[ \frac{2\pi \lambda^2}{\phi_0} B + \frac{1}{2} \log \frac{\phi_0}{2\pi \lambda^2 B} - \frac{1}{2}(1 - \gamma) \right], \quad (10)$$

$$\bar{f} \left( \frac{B}{H_{cr}} \right) = \frac{B}{H_{cr}} f \left( \theta_{\min} \left( \frac{B}{H_{cr}} \right) \right), \quad (11)$$

$$f(\theta) = \left( \frac{B}{H^*(t)} \right)^2 \sum'_{l,m} \frac{\sin^2 2\theta \cos 4\theta_{l,m}}{\left( (l+m)^2 \sin^2 \theta + (l-m)^2 \cos^2 \theta \right)^2} \quad (12)$$

$$+ \sum'_{l,m} \left( E_1 \left( \pi(l^2 \tan \theta + m^2 \cot \theta) \right) + \frac{(-1)^{l+m} + \exp(-\pi(l^2 \cot \theta + m^2 \tan \theta))}{\pi(l^2 \cot \theta + m^2 \tan \theta)} \right), \quad (13)$$

and

$$H^*(t) = \left( \frac{98\zeta(3)^2(2\pi)^3}{155a_1\zeta(5)(-\ln t)} \right)^{1/2} \frac{H_{c2}(t)}{\kappa} \simeq 5.64667(-\ln t)^{-1/2} \frac{H_{c2}(t)}{\kappa}. \quad (14)$$

By minimizing  $f(\theta)$  in  $\theta$ , we find the apex angle  $\theta_{\min}$  shown in Fig.3.  $\theta_{\min}$  increases first gradually and jumps to  $\pi/4$  at  $B = H_{cr}$  where

$$H_{cr} = 0.524(-\ln t)^{-1/2}\kappa^{-1}H_{c2}(t). \quad (15)$$

Earlier a similar  $\theta$ - $B$  curve is obtained within generalized London equation [9,10]. But here the arise of  $\theta$  to  $\pi/4$  is much steeper. Indeed contrary to [9,10], we predict that the transition is of the first order with a small dip in  $-M$  [11].

Unfortunately no experimental data on  $\theta_{\min}$  is available in the hole doped high  $T_c$  cuprates to our knowledge. However a very similar jump in  $\theta_{\min}$  as function of  $B$  as described here has been seen by SANS from  $\text{ErNi}_2\text{B}_2\text{C}$  at  $T = 3.1$  [15]. Although the superconductivity of borocarbides is believed to be of  $s$ -wave, both the existence of the rhombic vortex lattice in a magnetic field  $B$  parallel to the  $c$  axis seen by SANS [15,16] in  $\text{ErNi}_2\text{B}_2\text{C}$ ,  $\text{YNi}_2\text{B}_2\text{C}$  and  $\text{LuNi}_2\text{B}_2\text{C}$  and STM from  $\text{YNi}_2\text{B}_2\text{C}$  [17] suggest strongly that the underlying superconductivity is of  $d$ -wave. More recently the fourfold anisotropy in the uppercritical field in a magnetic field within the  $a$ - $b$  plane seen in  $\text{LuNi}_2\text{B}_2\text{C}$  [18] is again consistent with  $d$ -wave superconductivity [19,20].

To sum up we find that the vortex lattice is triangular as in a classical  $s$ -wave superconductor in a small magnetic field. As  $B$  increases the vortex lattice transforms first gradually and then rapidly into a square lattice at  $B = H_{cr}(t)$ . Also the final jump is extremely rapid, which is seen from a sharp dip in the magnetization.

Although the theoretical expression for  $H_{cr}(t)$  is obtained within the weak-coupling model and for  $t$  not too small (say  $t > 0.5$ ), from the temperature dependence of  $C(t)$  in [3], we expect the result should be valid semiquantitatively even for  $t \ll 1$ , if we replace  $-\ln t$  by  $1 - t$ . This allows us not only to estimate  $H_{cr}(0)$  in the optimally doped YBCO and Bi2212 crystals but also to construct the phase diagram for the rhombic vortex lattice. As readily seen from Fig.4 the rhombic lattice occupies the major part of the  $T$ - $B$  phase space of the vortex state unless disturbed by the vortex lattice melting.

Though  $H_{c2}(0)$  in YBCO or Bi2212 is not known for sure, we may assume  $H_{c2}(0) \sim 120\text{T}$  and  $300\text{T}$  for YBCO and Bi2212 respectively. Then making use of the fact  $\kappa \simeq 100$  for these systems, we expect that the vortex lattice transition takes place at  $B \sim 1\text{T}$  and  $3\text{T}$  for YBCO and Bi2212 respectively. The former is consistent with SANS and STM experiment in YBCO monocrystals mentioned in the beginning. For Bi2212, on the other hand, the melting transition appears to take place around  $B \sim 300$  Gauß for optimally, doped Bi2212 [8]. If it is really the case, it will be difficult to see the square lattice discussed here in Bi2212 unfortunately. On the other hand if this transition exists, it will be not difficult to identify by SANS, STM and even by thermodynamic measurement.

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## REFERENCES

- [1] D.J. Scalapino, *Phys. Rep.* **250** 329 (1995).
- [2] K. Maki and H. Won, *Ann. Phys.* **5** 320 (1996); *J. Phys.* I (France) **6** 2317 (1996).
- [3] H. Won and K. Maki, *Europhys. Lett.* **30** 421 (1995); *Phys. Rev. B* **53** 5927 (1996).
- [4] H. Won and K. Maki, *Physica B* (in press).
- [5] B. Keimer, W. Y. Shih, R. W. Erwin, J. W. Lynn, F. Dogan and I. A. Aksay, *Phys. Rev. Lett.* **73** 3459 (1994).
- [6] I. Maggio-Aprile, Ch. Renner, A. Erb, E. Walker and Ø. Fischer, *Phys. Rev. Lett.* **75** 2754 (1995).
- [7] D. E. Farrell “Transformation of the vortex solid” in “Physical Properties of high Temperature Superconductors IV”, edited by D.M.Ginsberg (World Scientific, Singapore 1994) pp7-60.
- [8] P. H. Kes, H. Pastoriza, T. W. Li, R. Cubitt, E. M. Forgan, S.L. Lee, M. Konczykowski, B. Khaykovich, D. Majer, D. T. Fuchs and E. Zeldov, *J. Phys.* I (France) **6** 2327 (1996).
- [9] M. Franz, I. Affleck and M. H. S. Amin, *Phys. Rev. Lett.* **79** 1555 (1997); I. Affleck, M. Franz and M. H. Sharifzaheh Amin, *Phys. Rev. B* **55** R704 (1997).
- [10] V. G. Kogan, P. Miranović, Lj. Dobrosavljević-Grujić, W. E. Pickett and D. K. Christen, *Phys. Rev. Lett.* **79** 741 (1997); V. G. Kogan, M. Bullock, B. Harmon, P. Miranović, Lj. Dobrosavljević-Grujić, *Phys. Rev. B* **55** R8693 (1997).
- [11] J. Shiraishi, M. Kohmoto and K. Maki, *preprint*.
- [12] H. Won and K. Maki, *Phys. Rev. B* **49** 1397 (1994).
- [13] N. Enomoto, M. Ichioka and K. Machida, *J. Phys. Soc. Jpn.* **66** 204 (1997).
- [14] A. L. Fetter, P. C. Hohenberg and P. Pincus, *Phys. Rev.* **147** 140 (1966); A. L. Fetter, *Phys. Rev.* **147** 153 (1966).
- [15] U. Yaron, P. L. Gammel, A. P. Ramirez, D. A. Huse, D. J. Bishop, A. I. Goldman, C. Stassis, P. C. Canfield, K. Mortensen and M. R. Eskildsen, *Nature (London)* **382** 236 (1996); M. R. Eskildsen, P. L. Gammel, B. P. Barber, A. P. Ramirez, D. J. Bishop, N. H. Andersen, K. Mortensen, C. A. Bolle, C. M. Lieber and P. C. Canfield, *Phys. Rev. Lett.* **78** 1968 (1997); **79** 487 (1997).
- [16] M. Yethiraj, D. McK. Paul, C. V. Tomy and E. M. Forgan, *Phys. Rev. Lett.* **78** 4849 (1997).
- [17] Y. De Wilde, M. Iavarone, U. Welp, V. Metlushko, A. E. Koshelev, I. Aranson and G. W. Crabtree, *Phys. Rev. Lett.* **78** 4273 (1997).
- [18] V. Metlushko, U. Welp, A. Koshelev, I. Aranson and G.W. Crabtree, *Phys. Rev. Lett.* **79** 1738 (1997).
- [19] H. Won and K. Maki, *Physica B* **199-200** 353 (1994); *Europhys. Lett.* **34** 453 (1996).
- [20] G-f. Wang and K. Maki, *preprint*.

## Figure Captions

Fig.1:  $|f(\mathbf{r})|$  in the  $x$ - $y$  plane. we used  $\epsilon = 1$  for the clarity of the figure, while  $\epsilon \simeq 0.1$  in the real situation.

Fig.2:  $\Phi(\mathbf{r}) = \text{phase of } f(\mathbf{r}) - \phi$  is shown in the  $x$ - $y$  plane for  $\epsilon = 1$ .

Fig.3: Appex angle  $2\theta_{\min}$  as a function of  $B/H_{cr}$  where  $2\theta_{\min} = 90^\circ$  and  $120^\circ$  correspond to the square lattice and the triangular lattice with hexagonal symmetry, respectively.

Fig.4: The phase diagram with the rhombic vortex lattice is shown schematically. In reality the rhombic lattice becomes stable around  $B = 10^{-2}H_{c2}(t)$ . Also “irr” means the irreducible line where the vortex lattice melts into the vortex liquid by the first order transition in the clean high  $T_c$  cuprate superconductor.

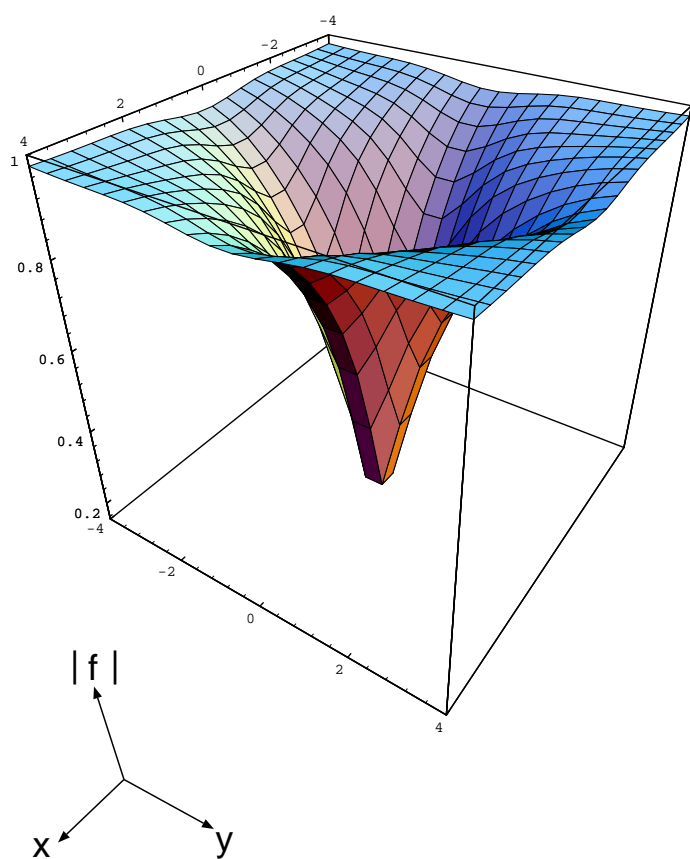
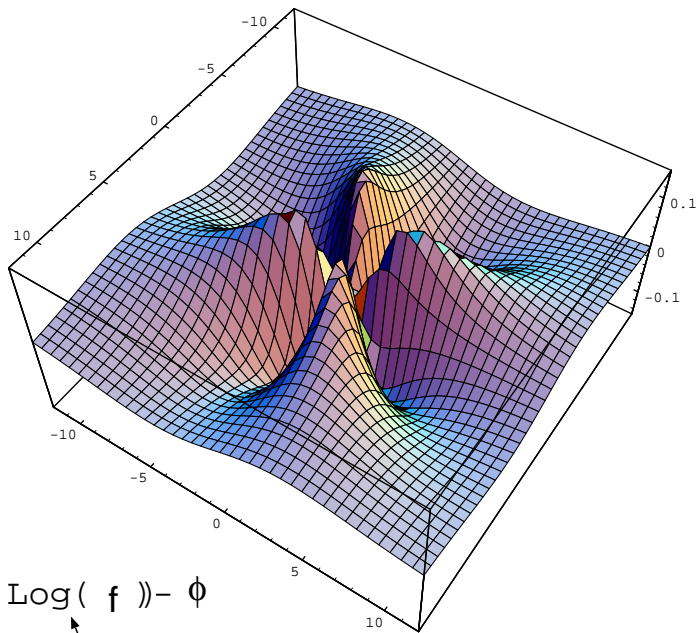
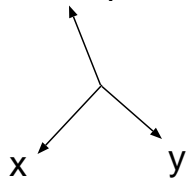


Fig.1



$\text{Im}(\text{Log}(f)) - \phi$



**Fig.2**



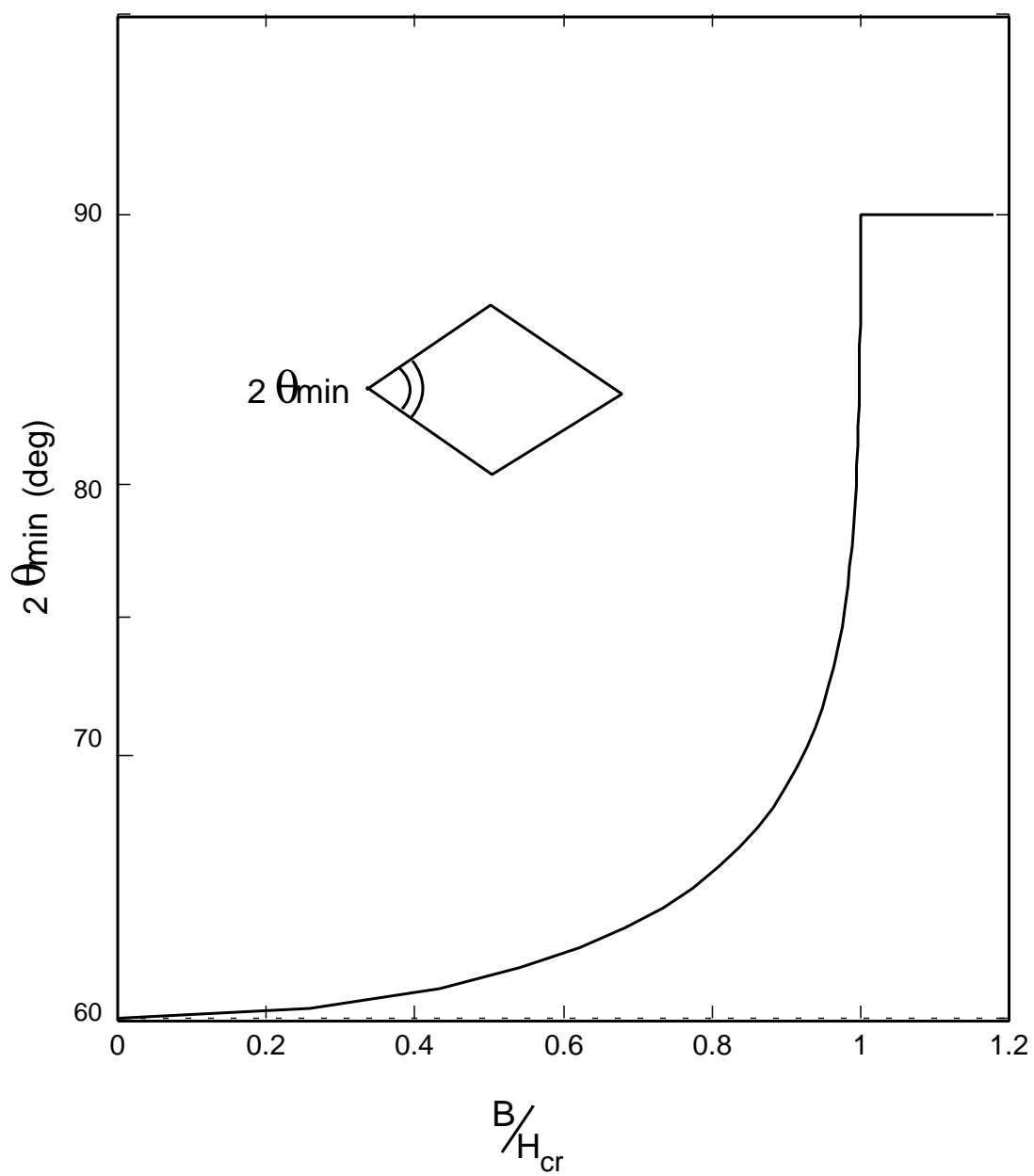


Fig.3

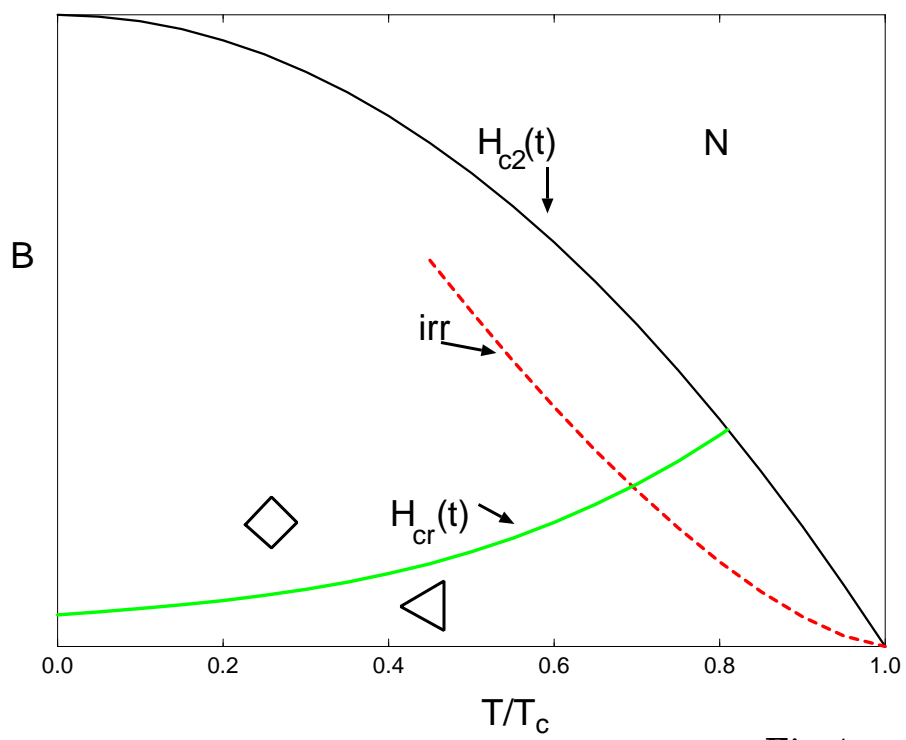


Fig.4